ABILITY OF VALUE AT RISK TO ESTIMATE THE RISK: HISTORICAL SIMULATION APPROACH

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Abstract: Each time an investor is investing on financial markets, he address himself the following question "What is the most I can lose?". One measure which help investors in identifying each stock's risk, it is represented by Value at Risk, which provides an answer in some reasonable bounds. This measure is widely used on financial markets, because it is easy to compute and implement. In the economic literature there are presented three ways for computing the Value at Risk, namely: variance-covariance approach, historical simulation approach and Monte Carlo simulation approach.
The aim of this paper is to estimate and compare Value at Risk for the three main financial markets' indices (NYSE composite – America, FTSE 100 – Europe and Nikkei 225 – Asia) based on two different periods of time: a period of financial distress and a period characterized by calm financial markets. In order to achieve this we will use the historical simulation for estimating the Value at Risk.
This research will provide proves regarding the performance of Value at Risk to estimate the risk, taking into account not only the confident level used in estimation, but moreover the difference in the period for which we estimate the risk: periods when financial markets are calm, and periods when financial markets are under distress.

Key words: Value at Risk, historical simulation, financial index, financial markets

JEL classification: G01; D53; D81

INTRODUCTION

The risk is one of the most important topic which affect the investments' strategies on financial markets. Of course, the term of risk can be associated with many kinds of events, but in general meaning this term refers to the probability of apparition of an undesired event which can lead to losses for investors. If we take into account the main causes which lead to risk apparition, we can classify the risk in several categories, such as: market risk, financial risk, credit risk, liquidity risk, legal risk, operational risk, country risk, and others.

Even if we are able to identify the main causes for risk, and after to classify the risk, it is hard to quantify the risk because there are many subjective aspects, which cannot be take into account by a simple formula. Despite this draw back, there were many researchers which tried to find ways to measure the risk. Risk management history dates back to 1945, when Leavens suggest a quantitative example for quantifying the risk. This was the starting point, because over the time, other authors improved the measure, or even proposed other kinds of quantifications.

The most used measure for risk quantification on financial markets is represented by Value at Risk, for which we cannot find in the literature a single definition. This aspect can be proved based on Wilson (1998) findings, who states that almost each financial institution has a unique name for its Value at Risk. For example, J.P. Morgan`s use Value at Risk (VaR) and Daily Earnings at Risk (DEaR); Bankers Trust are using Capital at Risk (CaR), while for other financial institutions we find the term Dollars at Risk (DaR) or Money at Risk (MaR). Despite these differences, all of these measures have three common elements: the maximum loss, a given probability and time horizon.

This paper, through which we want to analyze the ability of Value at Risk to capture the risk from financial market it is organize as follows: the first section will review the main papers which analyses the same topic, the second section is highlighting the methodological aspects used in the paper, the main data used in the analysis and some descriptive statistics

will be presented in section three, while the main results of the paper will be pointed in the section four. The last section will conclude the paper.

1. LITERATURE REVIEW

Value at Risk proved to be the simplest way to capture the risk of a financial instrument. This is the reason for which the specialized organization which are supervising the financial institutions, such as Basel Committee imposed to all banks to use this measure in order to perform regulatory capital calculations.

Over the time, the economic literature has outlined three ways of computing the Value at Risk:

- **Variance-Covariance approach** – In general terms, Value at Risk shows the probability to which a specific asset price is dropping under a specified limit. Due to this think, the Value at Risk may be computed based on probability distribution of potential values. This kind of approach it is very easy to implement, but the main draw bank is laying on the fact that sometimes can be very difficult to find the right probability distribution for the analyzed data.
- **Historical Simulation** – This approach estimates the Value at Risk based on the historical returns recorded for the analyzed financial instrument. That means we will take the historical prices for a financial asset, we compute the returns, and find the real distribution of the returns, based on which we compute the VaR.
- **Monte Carlo Simulation** – This approach is similar with the variance-covariance approach, but instead of computing the variance and covariance based on the assumed probability distribution for return, we will simulate this distribution based on the historical data.

Even if, Value at Risk is used by many financial and non-financial investors, this tool has some limitations, which were highlighted by several authors in their research. Beder (1995) emphasize that the liquidity risk, political, personnel and regulatory risks are not taking into account by Value at Risk. Going further, Linsmeier and Pearson (2000) states that the VaR estimates do not capture all information, because even if mathematical models are used to quantify the risk, there are many others factors which cannot be incorporated by these models, so the investors don't have all the information in order to manage the risk. Beside this, it seem that variance-covariance approach underestimates VaR. This is happening because you have to assume a probability distribution, and the type of the distribution is influencing a lot the final results. Going further, Sollis (2009) showed that historical simulation is altered by the sample size and even if Monte Carlo simulation seems a better solution, this approach also can be wrong due to the incorrect distribution assumption.

Through our previous researches (Anghelache et al., 2013; Oanea et al. 2013 and Zugravu et al. 2013) we analyzed the Value at Risk computed based on the variance-covariance approach, more specifically using the RiskMetrics methodology. Based on these papers, we emphasize the fact that the ability of VaR to capture the risk depends by several factors, such as: the probability distribution chosen for returns, the value of decay factor used by RiskMetric methodology, and the manner through which we compute this factor, and not in the end the sample size and data frequency used in the analysis.

Duffie and Pan (1997) tried to debate the main aspects regarding the risk, and the way to measure it. They emphasize the fact that the financial risk is very comprehensive, and includes the market risk which is the risk associated to the changes recorded for the price of a financial asset. Further, Hendricks (1996) analyzed twelve approaches used for modelling the Value at Risk, finding that all the approaches compute similar Value at Risk, because there is no statistically difference between the estimated VaR. Despite this, there are several limitation

for VaR estimation, because under normal distribution assumption for returns, there are occurring many extreme values, and moreover the changes recorded by the conditional volatility of the market can influence the ability of VaR estimates.

Another interesting article (Berkowitz and O'Brien, 2002) analyzed the performance of 5 banks' trading risk models. The main findings is that the modification in VaR estimates are positively correlated with all changes which appeared in the banks' profit or loss.

2. METHODOLOGY

According to de Vries (2000), in computing the Value at Risk we start with the set of the expected rates of return for the financial instrument, which we will note with ***R***. Further we can assume the fact that the set of the expected returns follow over a period of time the distribution function, presented by relation (1):

$$F(x) = \int_{-\infty}^{x} p(r)dr \quad \text{(1)}$$

, where $\boldsymbol{F} : \boldsymbol{R} \rightarrow [0, 1]$

where *p(r)* is the probability density function.

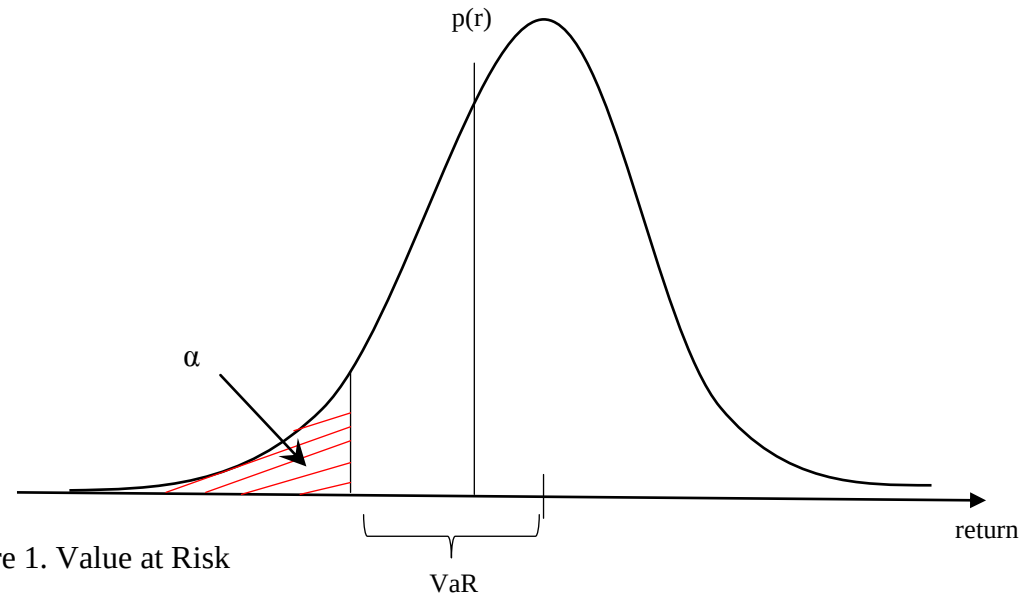


Figure 1. Value at Risk

Value at Risk for a period of time is defined as the maxim loss *L(t)* with probability (1-α), given by relation (2):

$$P(L(t) \le VaR) = 1 - \alpha \quad \text{(2)}$$

Because $P(L(t) \le VaR) = \tilde{F}(VaR)$, then VaR with respect to the selected period of time is the (1 − α) quartile of the variable loss:

$$VaR = \tilde{F}^{-1}(1-\alpha) \quad \text{(3)}.$$

In this article we will use the historical simulation, which is based on order statistics. For example, if we have 1,000 observations, the $VaR_{95\%}$ is nothing else than the 95 quartile of the n-days returns. When we estimate the Value at Risk based on the historical simulation, we have to take into account the confident level and the number of observation used in the estimation.

The next step after we compute the Value at Risk, based on the historical simulation, is represented by the testing of the estimation accurateness. As we already used in a previous research (Anghelache at al., 2013), the most frequently test used to test the estimation accurateness is conditional coverage test proposed by Christoffersen (1998). This test is a

joint test such that $LR_{CC} = LR_{UC} + LR_{IND}$, is $\chi^2_{(2)}$ distributed. Further we define a variable – I_t as:

$$I_t = \begin{cases} 1 & R_t < VaR_t \\ 0 & R_t \geq VaR_t \end{cases} \quad (4)$$

The conditional coverage test proposed by Christofersen is given by formula (5):

$$LR_{CC} = -2\ln\left(\frac{(1-\alpha)^{n_0}\alpha^{n_1}}{(1-\hat{\pi}_{01})^{n_{00}}\hat{\pi}_{01}^{n_{01}}(1-\hat{\pi}_{11})^{n_{10}}\hat{\pi}_{11}^{n_{11}}}\right) \sim \chi^2_{(2)} \quad (5)$$

where, n_{ij} is the number of observation with value i followed by j, $\pi_{ij} = \Pr(I_t = i \mid I_{t-1} = j)$ $(i, j = 0,1)$, $\hat{\pi}_{01} = \frac{n_{01}}{n_{00} + n_{01}}$, $\hat{\pi}_{11} = \frac{n_{11}}{n_{10} + n_{11}}$.

3. DATA AND DESCRIPTIVE STATISTICS

In this article we will estimate the Value at Risk using the historical simulation for three main representative financial markets: financial market from United States of America – *NYSE composite*, financial market from Europe – *FTSE 100* and financial market from Asia – *Nikkei 225*. For all of this we will estimate the Value at risk using 3 confidence interval, namely: 99%, 95% and 90%.

Table 1. Descriptive statistics for the three analysed indices' returns

Variable	Mean	Median	Max.	Min.	Std. Dev.	Skewness	Kurtosis
FTSE 100	0.0000	0.0000	0.0938	-0.0926	0.0124	-0.15	9.23
NIKKEI 225	-0.0001	0.0001	0.1323	-0.1211	0.0157	-0.42	9.25
NYSE Composite	0.0001	0.0006	0.1152	-0.1023	0.0131	-0.32	12.01

We use daily data for all the three indices, for the period January 1st, 2000 – December 31st, 2013. The date were obtain from the Yahoo Finance web site, and the analysis was done in the Gauss Light 8.0 program.

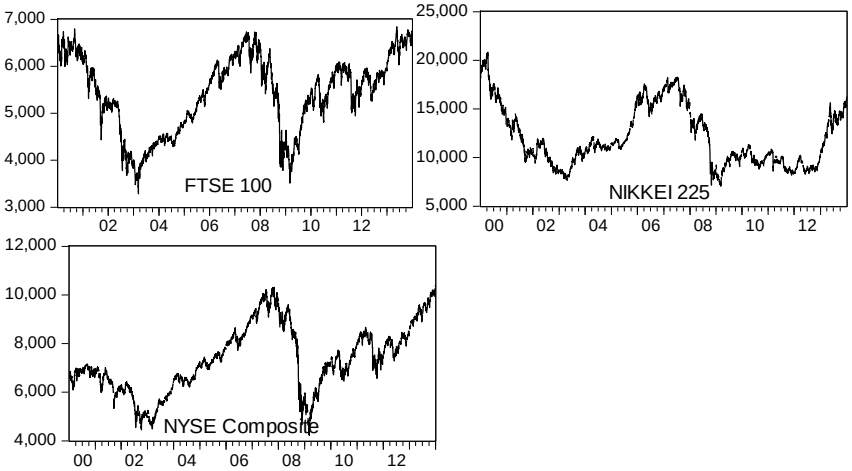


Figure 2. Price evolution for analyzed indices (2000 – 2013)

The main descriptive statistics for the daily returns of the analyzed indices are presented in the table 1. At a glance, we can see that all indices have recorded an average return of 0%, but in the same time we can see that the riskier is represented by the NIKKEI

225, because it has the higher standard deviation. Going further we present in the figure 2 the price evolution of these three indices. Based on this we can see that the financial crisis from 2008, had a powerful impact on all of them, so the main financial markets from the world were affected by the financial crisis started in U.S.A. in 2008.

4. RESULTS

In this article we will estimate the Value at Risk based on historical simulation. Moreover, we took in account 2 types of estimation regarding the sample size based on which we estimate the Value at Risk. The first way is to estimate the Value at Risk taking into account all historical data which are available starting with January 1st, 2000. This means that after we know a new return, we include it in the estimation sample, which is increasing on daily basis. The second way it is to use a fixed estimation period. So after we have a new return we include this value in the estimation period, but we will exclude the first value from the period, such as the number of observation to be constant (in our estimation we used a fix sample of daily data for 8 years).

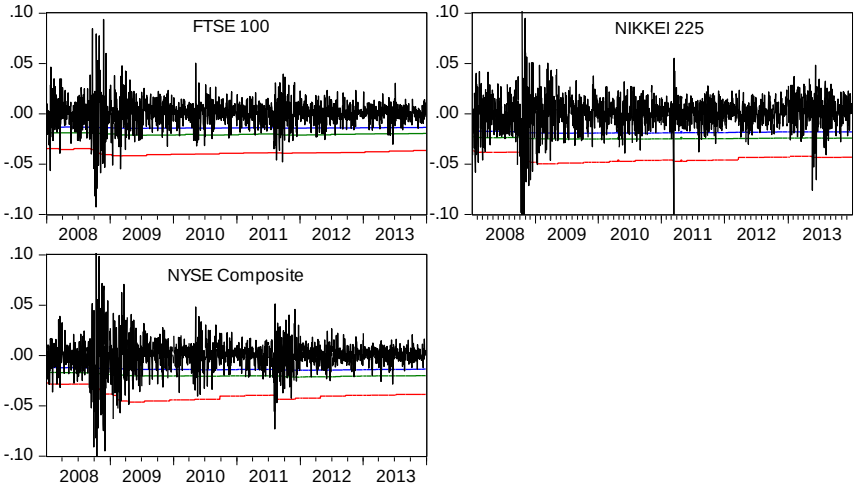
Table 2. Average Value at Risk – for period 2008 - 2013

Index	**FTSE 100**	**NIKKEI 225**	**NYSE Composite**
Full sample estimation			
VaR – 99%	-3.87%	-4.48%	-3.98%
VaR – 95%	-2.04%	-2.45%	-1.99%
VaR – 90%	-1.39%	-1.85%	-1.37%
Rolling window estimation			
VaR – 99%	-3.96%	-4.81%	-4.48%
VaR – 95%	-2.01%	-2.44%	-2.10%
VaR – 90%	-1.31%	-1.79%	-1.39%

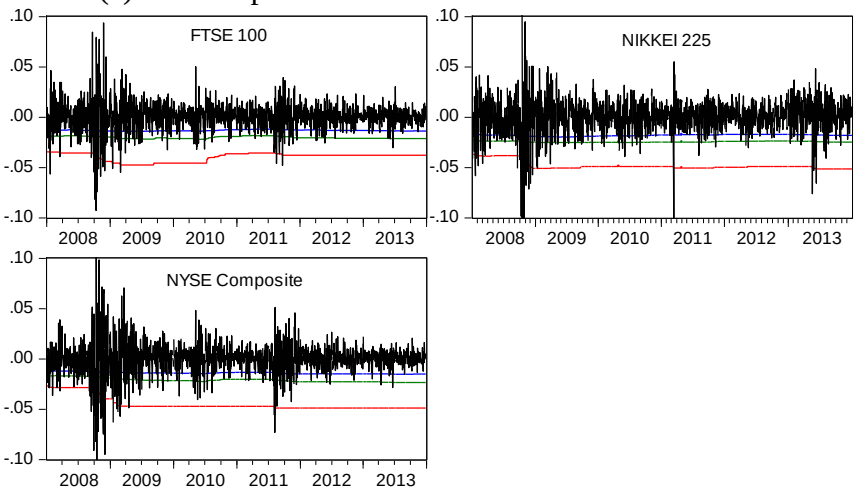
Based on these two procedures describe above, we estimate the Value at Risk, for the three analyzed indices, and taking into account 3 confident levels: 99%, 95% and 90%.

The average Value at Risk for the period 2008-2013 are presented in the table 2. At a first glance we can see that the lowest value at risk is estimated for Nikkei 225, and the highest one is recorded for FTSE 100, which means that for the analyzed period Asian financial market seems to be the riskier one, while the European financial market seems to be the most stable and less risky.

Moreover, we represent graphic the Value at Risk estimations for all indices in the figure 3, where we present the Value at Risk estimated based on the full sample, and also for the rolling window approach.



(a) full sample estimation



(b) rolling window estimation

Figure 3. VaR estimation for 2008 – 2013(VaR – 99%, VaR – 95%, VaR – 90%)

The efficiency of the approach used in the estimation of VaR were tested based on conditional coverage test sustained by Christoffersen (1998). Back-testing methodology was implemented by using the daily observation for year 2008 (period of financial distress) and year 2013 (period characterized by financial stability). Based on this we computed the conditional coverage test, for which the null hypothesis states that the model is correctly specified.

Table 3. Coverage test for year 2008

Index	**FTSE 100**	**NIKKEI 225**	**NYSE Composite**
Full sample estimation			
VaR – 99%	**7.66***	**0.77***	**2.01***
VaR – 95%	41.89	28.21	60.25
VaR – 90%	275.25	250.65	302.30
Rolling window estimation			
VaR – 99%	**7.66***	**0.77***	**2.01***
VaR – 95%	41.89	28.21	60.25
VaR – 90%	282.05	250.65	302.30

Note: The critical value for 99% is 9.210; Under H_0 the model is correct specified.

In table 4, we presented the test value for the Value at Risk estimated for the year 2008. At a first glance we can see that only with 99% confidence level the Value at risk is able to catch the risk existing on the market. Based on this result, we are able to say that the confidence level is a main factor which is influencing the ability of the Value at Risk, while the period selected is less important.

Table 4. Coverage test for year 2013

Index	**FTSE 100**	**NIKKEI 225**	**NYSE Composite**
Full sample estimation			
VaR – 99%	28.54	32.34	52.89
VaR – 95%	**8.98***	**0.29***	14.26
VaR – 90%	16.34	69.99	11.26
Rolling window estimation			
VaR – 99%	26.45	32.34	52.89
VaR – 95%	11.61	**0.29***	14.26
VaR – 90%	19.59	69.29	19.14

Note: The critical value for 99% is 9.210; Under H_0 the model is correct specified.

Further, we compute the coverage test for the year 2013, and the results are presented in the table 4. We can see that the most appropriate confident level is 95% for only 2 indices, FTSE 100 and NIKKEI 225.

5. CONCLUSIONS

The aim of this paper was to estimate and compare Value at Risk for the three main financial markets' indices, namely: NYSE composite –U.S.A. financial market, FTSE 100 – European financial market and Nikkei 225 – Asian financial market. Moreover, we estimate the VaR for two different periods of time: a period of financial distress – year 2008 and a period characterized by stable financial markets – year 2013. In order to achieve this we will use the historical simulation for estimating the Value at Risk, based on two approach regarding the estimation sample: full sample by taking into account all historical data which are available starting with January 1st, 2000, and fixed estimation period, based on rolling window method.

The efficiency of the approach used in the estimation of VaR were tested based on conditional coverage test sustained by Christoffersen (1998). Based on this, we saw that only with 99% confidence level the Value at Risk is able to catch the risk existing on the market in 2008. The main conclusion is that the confidence level is a main factor which is influencing the ability of the Value at Risk, while the period selected is less important.

The main conclusion of this paper, is the fact that in the period of financial distress, the Value at Risk is able to capture the risk existing on financial markets, only if we take into account the 99% confident level. This approach assume that the investors are risk averse and they will invest more prudential in time of financial distress.

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